Laboratory 6

Electronics Engineering 3210

The Fast Fourier Transform

Purpose:

This lab exercise introduces the student to the Fast Fourier Transform and some of the things that can be done with it.

Preliminary:

Write a title and short description of this lab on a new page of your lab book. Make an entry in the table of contents for this lab.

Part I. Fast Convolution

Write a MATLAB script to convolve the two functions in Figure 1 using the built-in functions **fft()** and **ifft()**. Assume a sampling rate of 10^6 Hz.

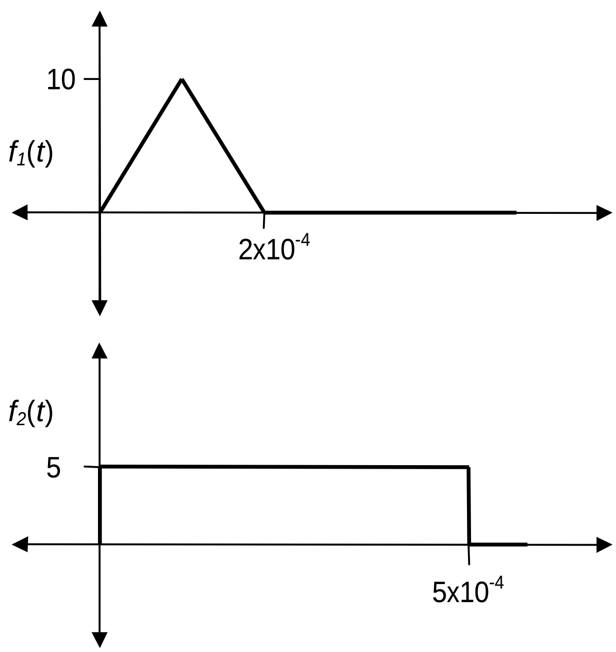

Figure 1. Input (Stimulus) Signal #1

Write code to compute and plot the same convolution using the built-**in conv()** function.

Part II. Signal Analysis.

In this exercise, you will use the FFT to analyze a signal to determine which frequency components it contains. You know that the signal is band-limited to 10kHz and the desired frequency resolution is 100Hz. This means that the signal must be sampled for a total of T_0=10ms. Find the minimum sampling rate that satisfies the Nyquist criterion and makes N_0 a power of 2. Record your calculations in your lab book.

Write a second MATLAB script that creates vectors t and w, both of which start at 0 and have length N_0. The time step should be the sampling interval, T_0/N_0 and the frequency step should be $\omega_0 = 2\pi/T_0$. Create an input signal consisting of a 1550Hz sinusoid, i.e.

```
f = To/No*cos(2*pi*1550*t);
```

Add code to label and plot f (against t). Write code to compute the FFT of f(t) and plot its magnitude (against w) as well. (Hint. Consider using the built-in **abs()** function)

Add code to apply the Hann window to f(t) before it is plotted and transformed, but comment it out for now:

```
% f = f .* (0.5-0.5*cos(2*pi*t/To));
```

Part III. System Response

Recall the system we used in Labs 3-5, shown below for convenience:

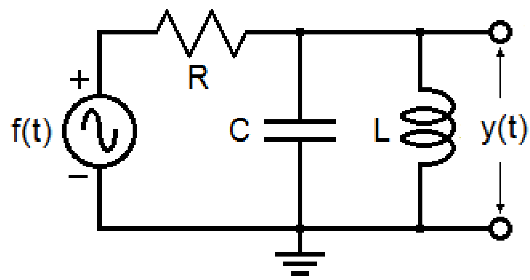

Figure 2. Series/Parallel Resonant Circuit

Let R = 1kΩ, C = 0.033μF and L = 1mH. Recall that the transfer function of this system is given by:

$$H(s) = \frac{s}{RCs^2 + s + \frac{R}{L}}$$

Find the essential bandwidth of this system. Make sure to include all your calculations in your lab book. (Hint: the maximum magnitude of $H(j\omega)$ is 1, so the frequency that $H(j\omega)$ drops to, say, 2% of its maximum value can be obtained by solving $|H(j\omega)|^2 = 0.02^2$.) Calculate the Nyquist frequency and Nyquist interval. Assume frequency resolution of 1000Hz (e.g. T_0 = 1ms). Find the minimum value for N_0 that will satisfy the Nyquist criterion.

Now choose a new, larger, N_0 that is a power of 2. Decide if you wish to sample faster or sample over a larger period, then calculate a new T_0 (period) and T (sampling interval).

Write a MATLAB script that creates a vector for $H(\omega)$ in a format suitable for the FFT. The first half of the vector should be $H(k\omega_0)$, $0 \le k < N_0/2$, and the second half should be

$H((k-N_0)\omega_0)$, $N_0/2 < k < N_0$. The value at $k = N_0/2$ should be $H(N_0\omega_0/2) + H(-N_0\omega_0/2)$ or $2Re(H(N_0\omega_0/2))$. One way to do this in MATLAB is shown in Figure 3.

```
R = 1000;
C = 3.3E-8;
L = 1.0E-3;
Hw = @(w) (1j*w)./(-R*C*w.*w+1j*w+R/L);
H = [Hw(0:Wo:Wo*(No/2)),Hw(-Wo*(No/2-1):Wo:-Wo)];
H(No/2) = 2*real(H(No/2));
```

Figure 3. MATLAB code to prepare H(ω) for use in the FFT

Add code to your script to create a vector of length N_0 for the time-domain signal f(t) shown in Figure 4 below.

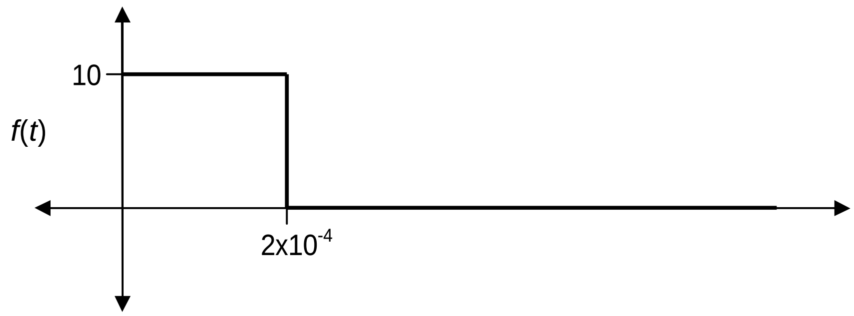

Figure 4. Input (Stimulus) Signal

This vector can be created (approximately) in MATLAB using the code below:

```
f = 10*[ 0.5, ones(1,floor(2E-4/T + 0.5)-1), 0.5 ];
f = [ f, zeros(1,No-length(f)) ];
```

Finally, write code that uses the built-in functions **fft()** and **ifft()** to compute F(ω) and the response of this system, y(t), respectively.

Procedure:

Part I. Fast Convolution

Run your first MATLAB script and plot the convolution $f_1(t) * f_2(t)$ obtained using the FFT. Repeat the procedure using the convolution of f_1 and f_2 using conv(). Verify that they are the same. Affix graph(s) to your lab book showing both results.

Part II. Signal Analysis

Run your second MATLAB script and make sure the FFT shows two distinct spikes, the first of which is at $\omega = 2\pi(1550)$. Affix a copies of graphs for f(t) and F(ω) in your lab book.

Uncomment the line that applies the Hann window and repeat the process. Make note in your log book the differences you observe between this F(ω) and the one without the Hann window.

Part III. System Response

Run your third MATLAB script and verify that the system response, y(t) is essentially the same as the one you obtained in Lab 5. Affix a copy of your graph of y(t) to your lab book.

Record any additional observations you have made, and write a conclusion in your lab book that summarizes what you have observed or discovered.